

# Problems with the perturbative QCD interpretation of HERMES data on semi-inclusive lepto-production of pions

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## Abstract

A theoretical analysis has been performed of the HERMES semi-inclusive deep inelastic data on the difference between  $\pi^+$  and  $\pi^-$  multiplicities. It turns out that the application of a standard perturbative QCD analysis, using, as usual, isospin symmetry for the fragmentation functions, leads to a poor description of the deuteron data at small  $z$  values. If one allows a breaking of isospin invariance, a good fit to both proton and deuteron data can be achieved for all measured  $z$ , but the level of isospin violation, especially at small values of  $z$ , is simply not credible. We suspect that the problem is a consequence of using the factorized QCD treatment in a kinematic region where it is unjustified.

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# 1 Introduction

Semi-inclusive deep inelastic production of hadrons in lepton-nucleon collisions plays a key role in the determination of polarized sea quark densities. In the perturbative QCD approach, valid in the kinematic region of large momentum transfer to the nucleon, the cross-section to produce a hadron  $h$  is expressed, in leading order (LO), as a product of parton densities  $q(x)$ , functions of Bjorken- $x$ , and fragmentation functions  $D_q^h(z)$ , reflecting the probability of a quark  $q$  to fragment into the hadron  $h$  carrying a fraction  $z$  of its energy. This factorized form is modified into a convolution when going beyond LO and, of course, the parton densities and fragmentation functions (FFs), develop a slow logarithmic dependence on the scale  $Q^2 = -q^2$ , where  $q^\mu$  is the 4-momentum transfer from the lepton to the nucleon.

In studying the latest HERMES data on  $\pi^\pm$  production on protons and deuterons [1] we have found that the application of the standard perturbative QCD analysis to the differences of the multiplicities  $M_{p(d)}^{\pi^+-\pi^-}(x, Q^2, z)$  using, as usual, isospin invariance for the FFs, leads to a poor description of the data at low  $z$ . Relaxing the demand for isospin invariance leads to violations of isospin conservation at a level which is simply not credible. The violation is biggest at small values of  $z$  and this has led us to suspect that the apparent violations are an artifact of using the factorized QCD treatment in a kinematic region where it is unjustified. Indeed, many years ago, Berger [2] proposed criterium for delineating the kinematic regions where the standard approach should be valid, and we have found that for the HERMES data, the data bin corresponding to the smallest values of  $z$  i.e.  $0.2 \leq z \leq 0.3$  lies outside Berger's safe region, as will be explained in detail in the paper.

## 2 QCD treatment of hadron multiplicities

The multiplicities  $M_{p(d)}^\pi(x, Q^2, z)$  of pions using a proton (deuteron) target are defined as the number of pions produced, normalized to the number of DIS events, and can be expressed in terms of the semi-inclusive cross section  $\sigma_{p(d)}^\pi$  and the inclusive cross section  $\sigma_{p(d)}^{DIS}$ :

$$\begin{aligned} M_{p(d)}^\pi(x, Q^2, z) &= \frac{d^3 N_{p(d)}^\pi(x, Q^2, z)/dx dQ^2 dz}{d^2 N_{p(d)}^{DIS}(x, Q^2)/dx dQ^2} = \frac{d^3 \sigma_{p(d)}^\pi(x, Q^2, z)/dx dQ^2 dz}{d^2 \sigma_{p(d)}^{DIS}(x, Q^2)/dx dQ^2} \\ &= \frac{(1 + (1 - y)^2)2x F_{1p(d)}^\pi(x, Q^2, z) + 2(1 - y)x F_{Lp(d)}^\pi(x, Q^2, z)}{(1 + (1 - y)^2)2x F_{1p(d)}(x, Q^2) + 2(1 - y)F_{Lp(d)}(x, Q^2)}. \quad (1) \end{aligned}$$

In Eq. (1)  $F_1^\pi, F_L^\pi$  and  $F_1, F_L$  are the semi-inclusive and the usual nucleon structure functions, respectively.  $F_1^\pi$  and  $F_L^\pi$  are expressed in terms of the unpolarized parton densities and fragmentation functions, while  $F_1$  and  $F_L$  are given by the unpolarized parton densities.

It turns out that the pion multiplicities are measured with enough precision so that one can directly extract from their differences  $M_{p(d)}^{\pi^+-\pi^-}(x, Q^2, z)$  the nonsinglet FFs  $D_q^{\pi^+-\pi^-}(z, Q^2) = (D_q^{\pi^+} - D_q^{\pi^-})(z, Q^2)$ . Knowledge of these is important to test whether the additional assumptions for the favored and unfavored FFs which are usually made in the QCD analyses of the multiplicities  $M_{p(d)}^{\pi^+}(x, Q^2, z)$  and  $M_{p(d)}^{\pi^-}(x, Q^2, z)$ , are or are not correct.

Using the charge conjugation invariance of the strong interactions in the case of the pion fragmentation functions

$$D_q^{\pi^+-\pi^-} = -D_{\bar{q}}^{\pi^+-\pi^-}, \quad D_g^{\pi^+-\pi^-} = 0 \quad (2)$$

and the assumption  $s(x, Q^2) = \bar{s}(x, Q^2)$  for the strange unpolarized parton densities, the following expressions for the proton and deuteron semi-inclusive structure functions hold in NLO QCD [3]<sup>1</sup>:

$$2F_{1p}^{(\pi^+-\pi^-)}(x, Q^2, z) = \frac{1}{9}(4u_v \otimes D_u^{\pi^+-\pi^-} + d_v \otimes D_d^{\pi^+-\pi^-}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi} C_{qq}^1), \quad (3)$$

$$F_{Lp}^{(\pi^+-\pi^-)}(x, Q^2, z) = \frac{\alpha_s(Q^2)}{2\pi} \frac{1}{9}(4u_v \otimes D_u^{\pi^+-\pi^-} + d_v \otimes D_d^{\pi^+-\pi^-}) \otimes C_{qq}^L, \quad (4)$$

$$2F_{1d}^{(\pi^+-\pi^-)}(x, Q^2, z) = \frac{1}{18}(u_v + d_v) \otimes (4D_u^{\pi^+-\pi^-} + D_d^{\pi^+-\pi^-}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi} C_{qq}^1), \quad (5)$$

$$F_{Ld}^{(\pi^+-\pi^-)}(x, Q^2, z) = \frac{\alpha_s(Q^2)}{2\pi} \frac{1}{18}(u_v + d_v) \otimes (4D_u^{\pi^+-\pi^-} + D_d^{\pi^+-\pi^-}) \otimes C_{qq}^L, \quad (6)$$

where  $u_v(x, Q^2)$  and  $d_v(x, Q^2)$  are the valence unpolarized parton densities,  $C_{qq}^1(x, z, Q^2)$  and  $C_{qq}^L(x, z, Q^2)$  are the Wilson coefficient functions [4]. Note also that the arguments  $(x, Q^2)$  for the parton densities and  $(z, Q^2)$  for the FFs in the equations above are omitted. The remarkable properties of these semi-inclusive structure functions are that the gluon fragmentation function does not contribute into the structure functions themselves, nor to the  $Q^2$  evolution of the nonsinglet quark fragmentation functions  $D_q^{\pi^+-\pi^-}(z, Q^2)$ .

According to isospin  $SU(2)$  symmetry

$$D_d^{\pi^+-\pi^-}(z, Q^2) = -D_u^{\pi^+-\pi^-}(z, Q^2) \quad (7)$$

and then Eqs. (3-6) take the following simple form:

$$2F_{1p}^{(\pi^+-\pi^-)}(x, Q^2, z) = \frac{1}{9}(4u_v - d_v) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi} C_{qq}^1) \otimes D_u^{\pi^+-\pi^-}, \quad (8)$$

$$F_{Lp}^{(\pi^+-\pi^-)}(x, Q^2, z) = \frac{\alpha_s(Q^2)}{2\pi} \frac{1}{9}(4u_v - d_v) \otimes C_{qq}^L \otimes D_u^{\pi^+-\pi^-}, \quad (9)$$

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<sup>1</sup>In this paper formulas are presented for the semi-inclusive cross section. Note that a factor 1/2 is missing in the formulas for the deuteron target.

$$2F_{1d}^{(\pi^+-\pi^-)}(x, Q^2, z) = \frac{1}{6}(u_v + d_v) \otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi} C_{qq}^1\right) \otimes D_u^{\pi^+-\pi^-}, \quad (10)$$

$$F_{Ld}^{(\pi^+-\pi^-)}(x, Q^2, z) = \frac{\alpha_s(Q^2)}{2\pi} \frac{1}{6}(u_v + d_v) \otimes C_{qq}^L \otimes D_u^{\pi^+-\pi^-}, \quad (11)$$

where the unpolarized valence quark densities  $q_v(x, Q^2)$  and the nonsinglet fragmentation function  $D_u^{\pi^+-\pi^-}(z, Q^2)$  satisfy the NLO QCD evolution equations.

In LO QCD approximation the longitudinal proton and deuteron structure functions  $F_{Lp(d)}^{(\pi^+-\pi^-)}$ ,  $F_{Lp(d)}$  are equal to zero and we obtain for the differences of the pion multiplicities:

$$M_p^{\pi^+-\pi^-}(x, Q^2, z) = \frac{(4u_v - d_v)(x, Q^2) D_{u_v}^{\pi^+}(z, Q^2)}{[4(u + \bar{u}) + d + \bar{d} + 2s](x, Q^2)}, \quad (12)$$

$$M_d^{\pi^+-\pi^-}(x, Q^2, z) = \frac{3(u_v + d_v)(x, Q^2) D_{u_v}^{\pi^+}(z, Q^2)}{[5(u + \bar{u} + d + \bar{d}) + 4s](x, Q^2)}, \quad (13)$$

where the notation  $D_{u_v}^{\pi^+}$  is used for the nonsinglet fragmentation function

$$D_{u_v}^{\pi^+} \equiv D_u^{\pi^+} - D_{\bar{u}}^{\pi^+} = D_u^{\pi^+-\pi^-}. \quad (14)$$

So, if isospin symmetry holds we have two independently measured quantities, the multiplicities  $M_p^{\pi^+-\pi^-}$  and  $M_d^{\pi^+-\pi^-}$  for the one nonsinglet fragmentation function  $D_{u_v}^{\pi^+}(z, Q^2)$  which can thus be determined in LO QCD directly and independently from the data at measured values of  $Q^2$  without using an input parametrization for  $D_{u_v}^{\pi^+}(z, Q^2)$  and its  $Q^2$  evolution.

## 3 Results

In our analysis we have used the  $[Q^2, z]$  presentation of the HERMES proton and deuteron data on pion multiplicities [1], corrected for exclusive vector meson production. The pion multiplicities are given for 4 z-bins [0.2-0.3; 0.3-0.4; 0.4-0.6; 0.6-0.8] as functions of  $Q^2$ . Note that to any measured value of  $Q^2$  the corresponding value of the Bjorken variable  $x$  is attached. The total number of the  $\pi^+$  and  $\pi^-$  data points for the proton and deuteron targets is 144, 72 for  $\pi^+$  and 72 for  $\pi^-$  data. So, there are 36 data points (9 for every z-bin) for the multiplicity  $M_p^{\pi^+-\pi^-}(x, Q^2, z)$ , as well as for  $M_d^{\pi^+-\pi^-}(x, Q^2, z)$ .

### 3.1 Isospin SU(2) symmetry

Our results on the fragmentation function  $D_{u_v}^{\pi^+}(z, Q^2)$  extracted from the proton data using Eq. (12) and deuteron data using Eq. (13) are presented in Fig. 1, blue and red points, respectively. For the LO parton densities the CTEQ6l parametrization [5]

has been used. In the calculations of the errors presented in Fig. 1 only the statistical errors of the multiplicities have been taken into account. The extracted nonsinglet FFs should coincide within the errors. As seen from Fig. 1, they are not in agreement for the first  $z$ -bin, and partially for the second one. The use of a different set of PDFs practically does not change the situation.

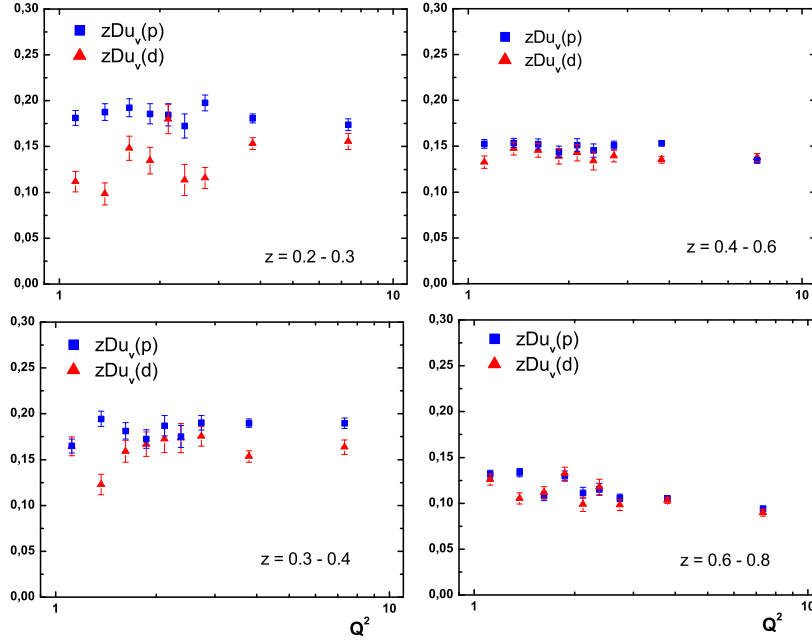


Figure 1: Comparison between the LO QCD nonsinglet fragmentation functions  $zD_{u_v}^{\pi^+}(z, Q^2)$  extracted from the differences of the pion multiplicities. Blue (red) points correspond to the data using a proton (deuteron) target, respectively.

We have performed the same analysis in NLO QCD using for the nonsinglet semi-inclusive structure functions Eqs. (8-11) and the NLO QCD expressions for the usual nucleon structure functions ( $F_1$ ,  $F_L$ ). For the unpolarized PDFs the NLO MRST'02 set [6] was used. In the NLO case one can not extract the nonsinglet fragmentation function  $D_{u_v}^{\pi^+}(z, Q^2)$  directly from the data, and we have to parametrize it at some fixed value of  $Q^2$ . The following parametrization for  $D_{u_v}^{\pi^+}(z, Q^2)$  at  $Q_0^2 = 1 \text{ GeV}^2$  was used:

$$zD_{u_v}^{\pi^+}(z, Q_0^2) = A_u z^{a_u} (1 - z)^{b_u} [1 + \gamma_u (1 - z)^{\delta_u}], \quad (15)$$

where the parameters  $\{A_u, a_u, b_u, \gamma_u, \delta_u\}$  are free parameters to be determined from the fit to the data. The double Mellin transform technique [7] has been used to calculate the nonsinglet semi-inclusive structure functions Eqs. (8-11) from their moments.

We have found that using only the very tiny statistical errors one can not achieve a satisfactory description of the data. We obtain the following values for  $\chi^2$  per point:  $\chi^2(p)/Nrp = 2.26$  from the fit to proton data,  $\chi^2(d)/Nrp = 1.96$  from the fit to deuteron data, and  $\chi^2(p)/Nrp = 5.01$ ,  $\chi^2(d)/Nrp = 6.49$  from the *combined* fit to the proton and deuteron data on the differences of pion multiplicities  $M_N^{\pi^+-\pi^-}(x, Q^2, z)$ , ( $N = p, d$ ). We would like to mention, however, that the statistical errors for the HERMES data are between two and three times smaller than the systematic ones. So, to obtain reasonable results from the fits to the data, the systematic errors in this case have to be taken into account.

In Fig. 2 the best fit curves (solid lines) of our NLO QCD *combined* fit to the proton and deuteron data on the differences of pion multiplicities using the total errors are compared with the data. The best fit curves (dashed lines) obtained from the separate fits to the proton and deuteron data are also presented. The numerical results for the

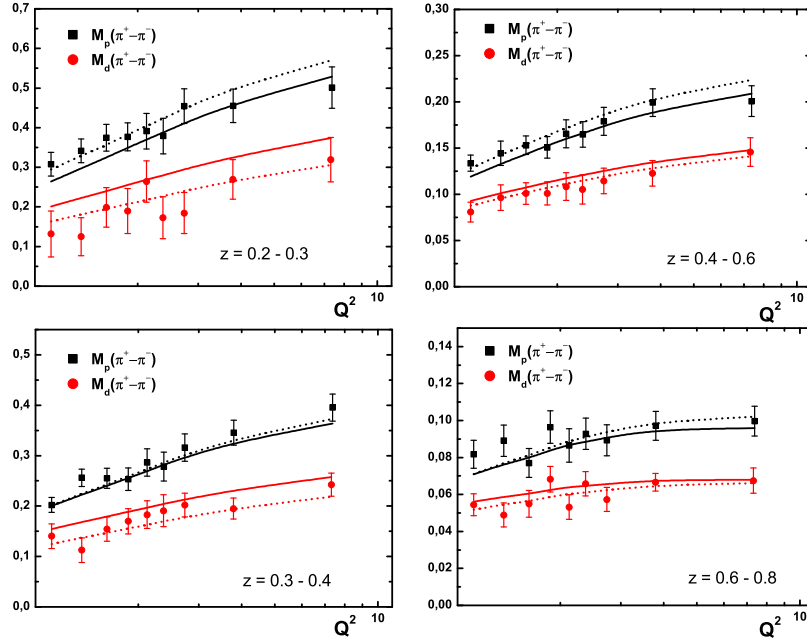


Figure 2: Comparison between the data and the best fit curves (black and red solid lines) obtained from a combined NLO QCD fit to the proton and deuteron data on the differences of the pion multiplicities. The dashed curves correspond to the separate fits to the proton and deuteron data. The errors of the data are combined statistical and systematic.

combined fit are given in Table 1. While in the case of separate fits to the proton and deuteron data an excellent description for *all*  $z$ -bins of the proton as well for the deuteron data is achieved:  $\chi^2(p)/DOF = 0.65$  and  $\chi^2(d)/DOF = 0.50$  (see the dashed

curves in Fig. 2), in the combined fit to the data the situation is different. A good description of the data is achieved except for the lower  $z$  bins,  $[0.2 < z < 0.3]$  and  $[0.3 < z < 0.4]$ , for the deuteron target, for which  $\chi^2/Nrp = 2.04$  and  $\chi^2/Nrp = 1.24$ , respectively (see Table 1).

**Table 1.** Results of the NLO QCD combined fit to the proton and deuteron data on the differences of pion multiplicities. The values of  $\chi^2$  per point are presented for all  $z$  bins as well as separately for each  $z$  bin.

Data	$N_{data}$	$\chi^2(proton)$	$\chi^2(deuteron)$
All $z$ -bins	36	0.79	1.19
$z_1$ -bin	9	0.96	2.04
$z_2$ -bin	9	0.84	1.24
$z_3$ -bin	9	0.59	0.59
$z_4$ -bin	9	0.79	0.89

One can see also in Fig. 2 that for these two bins the best fit curves (solid red lines) lie systematically higher than the central values of the data. Note that in the case of separate fits, it was enough to use 3 free parameters for the input parametrization of the nonsinglet structure function  $zD_{u_v}^{\pi^+}(z, Q_0^2)$  ( $\gamma = 0$  in Eq. (15)), while in the combined fit a better description of the data was achieved using all 5 free parameters. The

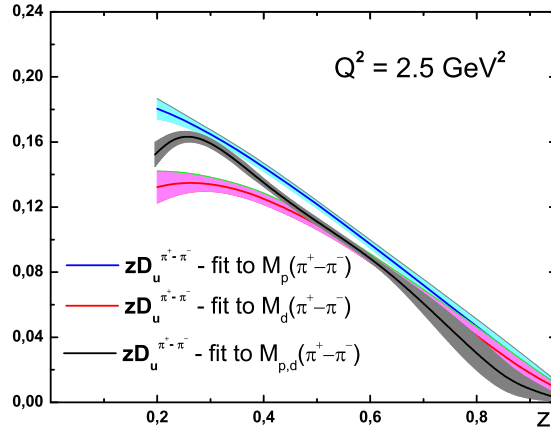


Figure 3: The nonsinglet fragmentation function  $zD_u^{\pi^+-\pi^-}(z)$  at  $Q^2 = 2.5 \text{ GeV}^2$  extracted from NLO QCD fits to a) proton, b) deuteron and c) proton and deuteron data on the differences of pion multiplicities.

extracted nonsinglet fragmentation function  $zD_u^{\pi^+-\pi^-}(z, Q^2)$  from the combined NLO QCD fit to the proton and deuteron data is presented in Fig. 3 as a function of  $z$  for  $Q^2 = 2.5 \text{ GeV}^2$  (black curve) together with its error band, and in Fig. 4 as a function

of  $Q^2$  at any fixed  $z$ -bin for the measured  $Q^2$  values. In both the figures it is compared with the nonsinglet FFs extracted from the separate fits to the proton (blue curves) and deuteron (red curves) data. In Fig. 4 the LO nonsinglet FFs extracted directly from the proton (blue points) and deuteron (red points) data, are also presented.

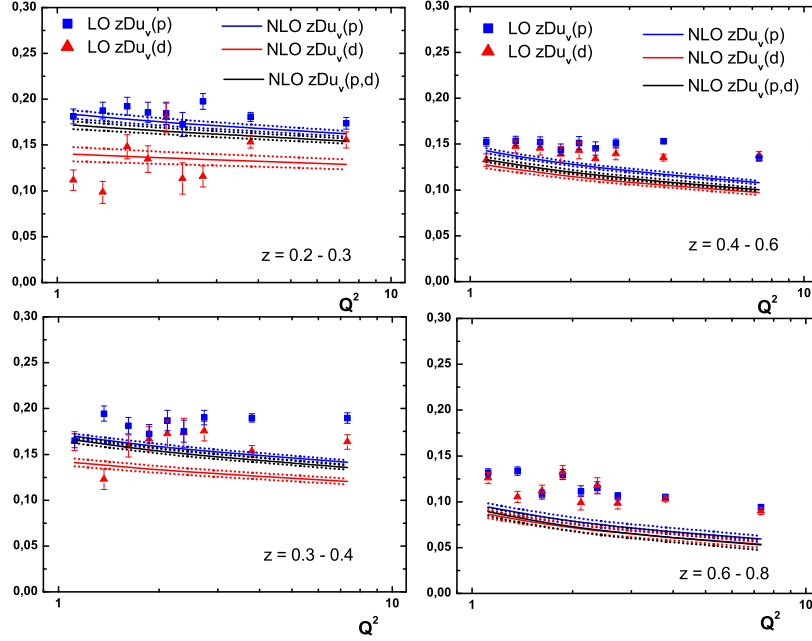


Figure 4: Comparison between the LO and NLO nonsinglet fragmentation functions  $D_{u_v}^{\pi^+}(Q^2)$  at the different  $z$ -bins. For details see the text.

So, one can conclude from our NLO results that the combined fit to the proton and deuteron data on the differences of the pion multiplicities confirm the observation found in the LO analysis: Assuming  $SU(2)$  symmetry for the pion fragmentation functions and applying the factorized QCD treatment to the HERMES  $[Q^2, z]$  data leads to problems with the description of the data for the lowest  $z$  bins. In addition, an important fact coming from the NLO analysis is that the problems are connected *only* with the deuteron data (see Table 1 and red curves in Fig. 2). We have no explanation of this point.

### 3.2 $SU(2)$ symmetry breaking

One way to try to achieve a better fit to the data in the low  $z$  bins too is to suppose that isospin  $SU(2)$  symmetry is broken. In this case Eq. (7) does not hold and there are two independent nonsinglet fragmentation functions  $D_u^{\pi^+-\pi^-}(z, Q^2)$  and  $D_d^{\pi^+-\pi^-}(z, Q^2)$  to



be determined simultaneously from the NLO combined fit to the proton and deuteron data, using for the semi-inclusive structure functions Eqs. (3-6). The new nonsinglet  $D_d^{\pi^+-\pi^-}(z, Q^2) \equiv D_{dv}^{\pi^+}(z, Q^2)$  is parametrized at  $Q_0^2 = 1 \text{ GeV}^2$  like  $D_{uv}^{\pi^+}$  (see Eq. (15)) with free parameters  $\{A_d, a_d, b_d, \gamma_d, \delta_d\}$ . The equality  $a_d = a_u$  for the parameters  $a_u$  and  $a_d$  has been used in the fit. The results of the fit are illustrated in Table 2 and Fig.5.

**Table 2.** Results of the NLO QCD combined fit to the proton and deuteron data on the differences of pion multiplicities when  $SU(2)$  symmetry is broken. The values of  $\chi^2$  per point are presented for all  $z$  bins as well as separately for each  $z$  bin. The values in the brackets correspond to  $SU(2)$ -symmetry fit.

Data	$N_{data}$	$\chi^2(proton)$	$\chi^2(deuteron)$
All $z$ -bins	36	0.82 (0.79)	0.50 (1.19)
$z_1$ -bin	9	0.86 (0.96)	0.83 (2.04)
$z_2$ -bin	9	0.77 (0.84)	0.44 (1.24)
$z_3$ -bin	9	0.81 (0.59)	0.14 (0.59)
$z_4$ -bin	9	0.85 (0.85)	0.60 (0.89)

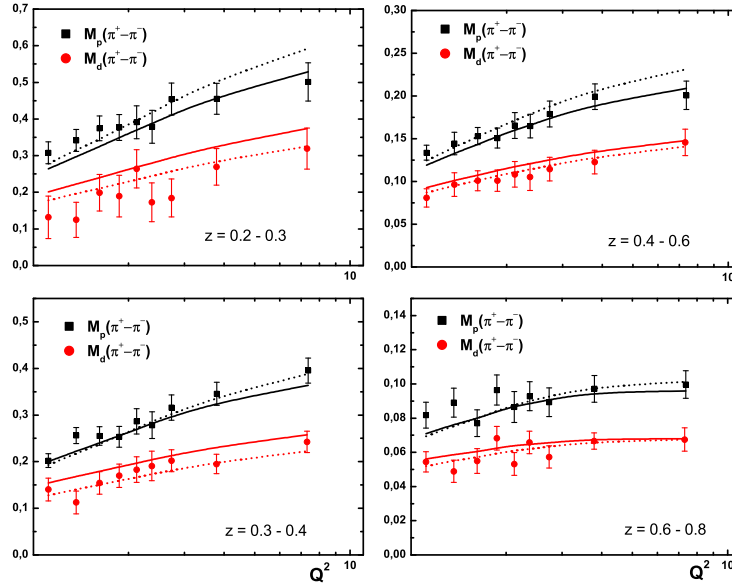


Figure 5: Comparison between the data and the best fit curves (black and red dashed lines) obtained from a combined NLO QCD fit to the proton and deuteron data on the differences of pion multiplicities without  $SU(2)$  symmetry. The solid curves correspond to the  $SU(2)$  symmetric fit to the same data.

As seen from Table 2 and Fig. 5, a significant improvement in the fit to the deuteron data for the first two  $z$ -bins is achieved and a good description of both the proton and deuteron data is obtained. However, as seen from Fig. 6, where the extracted nonsinglet FFs  $D_u^{\pi^+-\pi^-}(z, Q^2)$  and  $D_d^{\pi^+-\pi^-}(z, Q^2)$  are presented, the violation of  $SU(2)$  symmetry is at a level, especially for the values of  $z < 0.4$ , which is hardly credible. The relation between the two nonsinglets at  $Q^2 = 1 \text{ GeV}^2$  obtained from the fit to the data is:

$$D_d^{\pi^+-\pi^-}(z) = -2.18(1-z)^{0.46} D_u^{\pi^+-\pi^-}(z) \quad (16)$$

Remember that when  $SU(2)$  symmetry holds:  $D_d^{\pi^+-\pi^-}(z) = -D_u^{\pi^+-\pi^-}(z)$ .

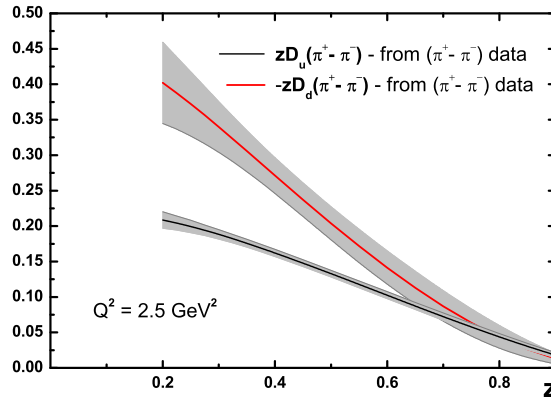


Figure 6: Nonsinglet fragmentation functions  $zD_u^{(\pi^+-\pi^-)}(z)$  and  $zD_d^{(\pi^+-\pi^-)}(z)$  at  $Q^2 = 2.5 \text{ GeV}^2$  together with their error bands extracted from a NLO QCD combined fit to proton and deuteron data on the differences of pion multiplicities.

## 4 Comments

We think that what appears as a non-credible violation of isospin symmetry could be a consequence of using the factorized QCD treatment of the data in kinematic region where it is unjustified. Indeed, according to Berger's phenomenological criterium [2] proposed a long time ago, there is a strong correlation between  $W$ , the invariant mass of the hadrons  $X$  produced in the fully inclusive lepton nucleon process  $l + N \rightarrow l + X$ , and the region  $z$ , where one can clearly separate the quark and target fragmentation effects, i.e. where the hard scattering and the hadronization factorize and the usual factorized QCD treatment is valid. It was shown in [2] that the smaller the values of  $z$ , the larger the values of  $W$  have to be for a clean separation between the current and target jets. In [8] plots (Fig. 2 and Fig. 3) are presented showing the  $z$ -values for which it is probably safe to use the factorized approach, for a quark fragmenting into different hadrons ( $\pi, K, N, \Lambda$ ), produced in SIDIS processes, for values of  $W = 5$

and  $W = 20$ . As seen from these plots, in the case of pions, values of  $W \geq 5 \text{ GeV}$  are needed for a clean separation of the quark and target fragmentation effects in the data, and an unambiguous extraction of the collinear fragmentation functions  $D_{q,\bar{q}}^{\pi^+}(z, Q^2)$  for  $z \geq 0.2$ . Keeping in mind that for the HERMES data on pion multiplicities in the region  $0.2 \leq z \leq 0.8$  the corresponding values of  $W$  are in the region  $4.1 < W < 4.5$ , one sees that Berger's criterium is not satisfied for the smaller values of  $z$ , and certainly for the first  $z$ -bin,  $0.2 \leq z \leq 0.3$ .

Finally, we would like to mention that in the  $SU(2)$ -symmetry case we have found good agreement between the nonsinglet  $D_u^{\pi^+-\pi^-}(z, Q^2)$  extracted directly from the data on the differences of the pion multiplicities  $M_{p(d)}^{\pi^+-\pi^-}(x, Q^2, z)$  and that obtained as a difference between the favored  $D_u^{\pi^+}$  and unfavored  $D_u^{\pi^-} = D_{\bar{u}}^{\pi^+}$  fragmentation functions extracted from our fit to  $M_{p,d}^{\pi^+}$  and  $M_{p,d}^{\pi^-}$  data [9]. The nonsinglets extracted in these two different ways are shown in Fig. 7 (note that the highest value of  $z$  for the data is 0.7). The agreement shown in Fig (7) confirms that the usual assumption about the unfavored fragmentation functions

$$D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = D_{\bar{u}}^{\pi^+} \quad (17)$$

made in our fit [9] to the  $M_{p,d}^{\pi^+}$  and  $M_{p,d}^{\pi^-}$  data, is acceptable, but at the same time the agreement is surprising given the problems reported earlier about fitting the deuteron data on the difference of pion multiplicities in the lowest  $z$  bins, when imposing isospin invariance.

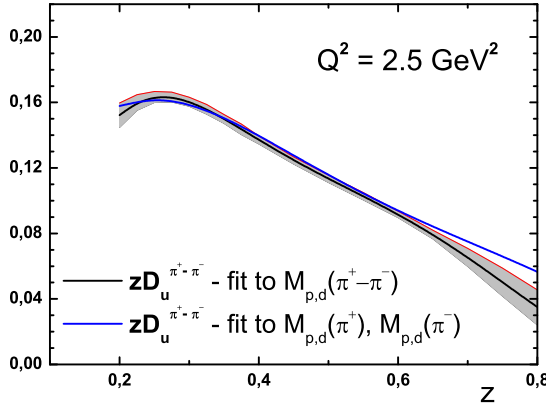


Figure 7: Comparison between nonsinglet fragmentation function  $zD_u^{(\pi^+-\pi^-)}(z)$  extracted from the fit to proton and deuteron data on the differences of the pion multiplicities and that one constructed from favored  $zD_u^{\pi^+}$  and unfavored  $zD_u^{\pi^-}$  fragmentation functions determined from a NLO combined fit to proton and deuteron data on pion multiplicities themselves.

Our NLO *favored* and *unfavored* pion FFs, extracted from the HERMES data on multiplicities  $M_{p,d}^{\pi^+}$  and  $M_{p,d}^{\pi^-}$  have been discussed in our paper [9] and compared to

those determined by HKNS (Hirai, Kumano, Nagai, Sudoh) [10] and DSS (de Florian, Sassot, Stratmann) [11] which were obtained respectively from the semi-inclusive  $e^+e^-$  annihilation data alone, and from the global fit to the semi-inclusive  $e^+e^-$  annihilation data, the data on single-inclusive hadron production in hadron-hadron collisions and the *unpublished* HERMES'05 data on semi-inclusive lepto-production of hadrons.

In Fig. 8 we compare our NLO nonsinglet fragmentation function  $zD_u^{(\pi^+-\pi^-)}(z)$  extracted *directly* from the HERMES data on the difference between  $\pi^+$  and  $\pi^-$  multiplicities to those of HKNS and DSS obtained as differences between their favored  $D_u^{\pi^+}$  and unfavored  $D_u^{\pi^-} = D_u^{\pi^-}$  NLO fragmentation functions. Note that in the DSS analysis  $SU(2)$ - symmetry was broken and Eq. (7) does not hold. That is why the nonsinglet fragmentation function  $-zD_d^{(\pi^+-\pi^-)}(z)$  for DSS is also presented in Fig. 8. As seen from Fig. 8, the discrepancy between the nonsinglets, is significant.

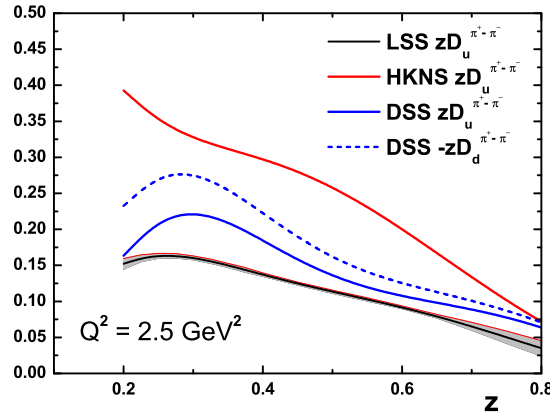


Figure 8: Comparison between our NLO nonsinglet fragmentation function  $zD_u^{(\pi^+-\pi^-)}(z)$  extracted from the fit to proton and deuteron data on the differences of the pion multiplicities and those of HKNS and DSS constructed from their favored  $zD_u^{\pi^+}$  and unfavored  $zD_u^{\pi^-}$  NLO fragmentation functions.

It is important to mention that the semi-inclusive  $e^+e^-$  annihilation data give no information how to disentangle quark  $D_q^h(z, Q^2)$  from anti-quark  $D_{\bar{q}}^h(z, Q^2)$  fragmentation, and only their sum  $D_q^h + D_{\bar{q}}^h$  can be determined from the data, while the important role of the semi-inclusive DIS processes is that they allow to separate  $D_q^h(z, Q^2)$  from  $D_{\bar{q}}^h(z, Q^2)$ . We think that this is the reason for the large difference between our nonsinglet FF and the HKNS one. As for the inconsistency between our and the DSS nonsinglet FFs, that is a consequence of the fact that the DSS group has used in their analysis the unpublished HERMES'05 data which are not consistent with the final HERMES data which we have used [1].

We feel it is important to mention a totally different issue which might be the source of the entire problem in the perturbative QCD description of the HERMES data, namely the fact that there appears to be an inconsistency between the two ways

HERMES has chosen to present their data. It appears to us that the  $[x, z]$  and the  $[Q^2, z]$  presentations are incompatible [9]. This could be a signal that maybe there is something wrong with the data.

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